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BRIEF COMMUNICATION

A NOTE ON THE EFFECTS OF BULK DENSITY VS INTERSTITIAL FLUID DENSITY IN STABILITY CONSIDERATIONS OF A SUSPENSION OVERLAIN BY A HEAVY FLUID

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1. INTRODUCTION

Consider a suspension of particles of density ρ_D in an interstitial fluid of density ρ_C. Let $\rho_D > \rho_C$, i.e. the particles are "heavy" and settle to the bottom in a gravity field (the subscripts C and D denote the continuous and dispersed "phases"). The bulk (or averaged) density is $\rho = \epsilon\rho_D + (1-\epsilon)\rho_C$, where ϵ is the volume fraction of the dispersed "phase", in general a function of space and time.

It is generally accepted—and consistent with experiments—that in stable situations the suspension can be treated as a regular single-phase fluid of density ρ, provided that ϵ is not exceedingly small, so that the number of particles in the system is large and the interparticle distance is much smaller than the dimensions of the container (Ungarish 1993).

Huppert *et al.* (1991) carried over that postulate about ρ to stability considerations. For the sake of explanation, this extension can be cast in the following way: suppose that a layer of suspension of bulky density ρ is overlain by a layer of fluid of density ρ_U in a gravity field; if $\rho_U \leqslant \rho$ the suspension domain is stable, and behaves as though the overlaying fluid does not exist. There is actually nothing special about this argument as long as the density of the interstitial fluid, ρ_C, is larger than ρ_U. However, Huppert *et al.* used this criterion for cases with $\rho_C < \rho_U$, and found good agreement with experiments. Additional confirmation was provided by Kerr & Lister (1992).

The underlying mechanism of the criterion and results of Huppert *et al.* is not evident. It is the purpose of this note to raise some pertinent questions and to suggest some possible explanations. This topic is worthy of investigation because: (a) it is related with the fundamental problem of the relevance of the bulk (averaged) properties of a suspension, and (b) the type of problems analysed by Huppert *et al.* are important in geophysical applications.

To give quantitative support to the discussion we perform some simple calculations with the typical parameters of the apparatus of Huppert *et al.* with "type 2" particles of radius $a = 12.5\ \mu$m and: $\rho_C = 1.000\ \mathrm{g\,cm^{-3}}$, $\rho_U = 1.070\ \mathrm{g\,cm^{-3}}$, $\rho_D = 3.22\ \mathrm{g\,cm^{-3}}$, viscosity $\nu = 0.01\ \mathrm{cm^2\,s^{-1}}$ (for all fluids), and zero surface tension. We shall use different values of ϵ as specified later, in the range of several percent. The section of the settling tank was $3 \times 19.5\ \mathrm{cm^2}$, the settling length was 20 cm, and the settling time was several min.

2. ANALYSIS

2.1. *The case $\rho > \rho_U$*

– *2.1.1. The suspension domain.* There is nothing special in the stability considerations when $\rho_U < \rho_C$. The settling particles, initially at $z = z_0$, leave behind a region of pure fluid, where $\epsilon = 0$ and the density is ρ_C, of increasing thickness

$$z_F(t) - z_0 = h(\epsilon) V_{\text{Stokes}} t. \qquad [1]$$

Here t is time, z is the coordinate in downward vertical direction, $h(\epsilon) \approx (1-\epsilon)^5$ is the hindrance function, $V_{\text{Stokes}} = (2/9)a^2(\rho_D - \rho_C)\mathbf{g}/\rho_C \nu$ is the classic settling velocity of the particle of radius a in the pure interstitial fluid whose viscosity is ν, and the subscript F denotes the particle–pure fluid interface ("Front"). Since the density of the pure fluid in the domain $z_0 < z < z_F(t)$ is larger than that of the overlaying fluid above z_0 the usual instability modes are bound to decay.

On the other hand, when $\rho_U > \rho_C$ the interface $z = z_0$ is inherently unstable, and we expect that the layers of fluid will quickly change places. This expectation is consistent with the experiments. However, the amazing thing is that the abovementioned instability mixing does not penetrate into the suspension fluid below the interface $z = z_F$ provided that $\rho > \rho_U$ [i.e. $(\rho_D - \rho_C)$ and ϵ are so big that their product exceeds the difference $(\rho_U - \rho_C)$]. The experimental occurrence of this surprising stability of the interface $z = z_F(t)$ and of the region below it, as opposed to the unstable convection above $z_F(t)$, was clearly reported by Huppert *et al.*: "a very sharp interface separated the convective region from an underlying sedimenting region . . ." and "the process of sedimentation in the lower layer is largely independent of the motion in the upper layer".

The dilemma comes from the fact that at the interface $z = z_F$ the lighter interstitial fluid is essentially overlain by the heavier fluid, as illustrated in figure 1. So, why is this interface stable? Actually, this question is relevant to the entire suspension below z_F because if the upper heavy fluid, U, starts replacing the light continuous fluid, C, below z_F this process is bound to propagate into the entire domain of unsettled suspension.

We suggest that this enhanced stability feature is a consequence of both the presence of the particles in the suspension and of their motion relative to the embedding fluid. First, these particles introduce a new length-scale, the interparticle distance e, and therefore cut off the usual instability modes for wavelengths larger than e. To verify this hypothesis we perform some simple calculations.

The interparticle distance can be estimated as (Ungarish 1993, section 2.2)

$$e = 2.0a\epsilon^{-1/3}. \qquad [2]$$

When the fluid C is overlain by heavier fluid U all perturbation modes are unstable, so the more relevant information concerns the wavelength of maximal growth, λ_{mg}, and, for a given λ, the exponent coefficient of growth, n (Chandrasekhar 1955); see figure 2.

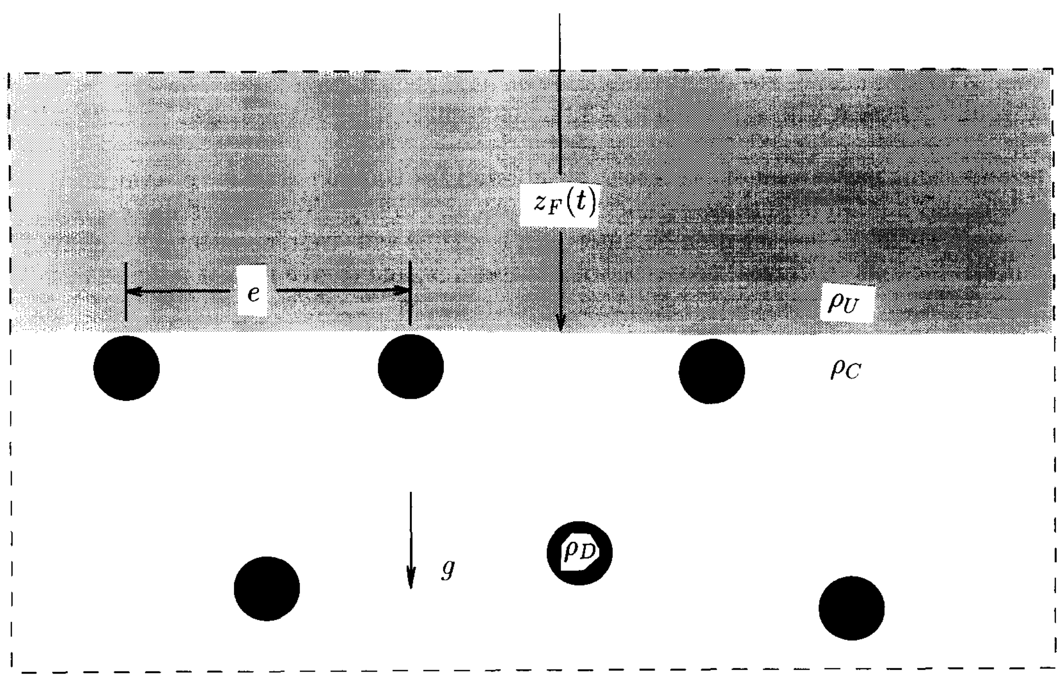

Figure 1. Enlargement of the suspension–pure fluid interface region when $\rho_U > \rho_C$. z_F is measured from the top wall, and $z_F(0) = z_0$.

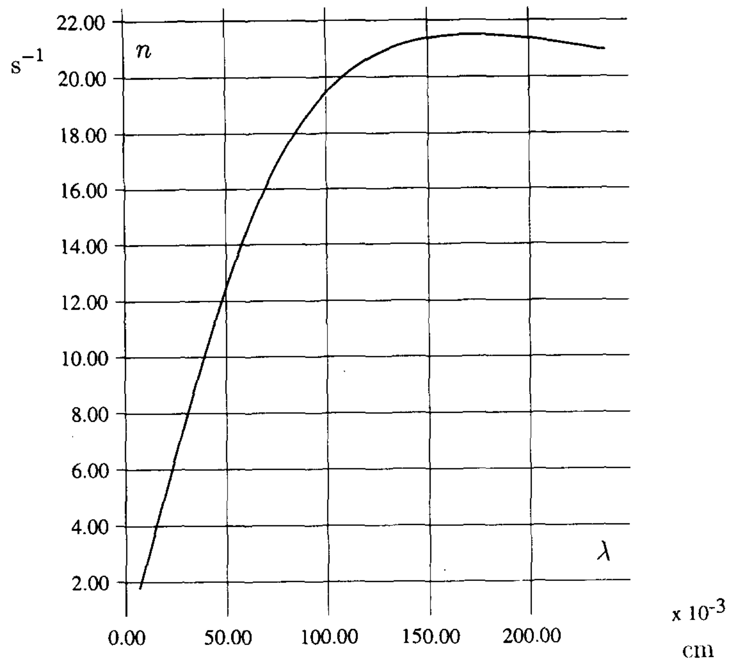

Figure 2. Viscous mode, n vs λ, for semi-infinite layers $\rho_C = 1.000\ \text{g cm}^{-3}$, $\rho_U = 1.070\ \text{g cm}^{-3}$, $\nu = 0.01\ \text{cm}^2\ \text{s}^{-1}$.

Indeed, in the abovementioned experiments λ_{mg} was much smaller than the dimensions of the container but larger by factors of at least 10 than e. To be more specific, we take $\epsilon = 0.04$ (in which case the suspension density is $\rho = \epsilon\rho_D + (1-\epsilon)\rho_C = 1.089$). We obtain: $e = 0.0073$ cm, $\lambda_{mg} = 0.17$ cm, $n_{mg} = 22\ \text{s}^{-1}$. On the other hand, the viscous mode with $\lambda = e = 0.0073$ cm corresponds to $n = 2\ \text{s}^{-1}$.

Evidently, the development of the unstable modes driven by the density difference $(\rho_U - \rho_C)$ is drastically hindered by the interparticle length scale, e.

However, the viscous mode with $\lambda = e$ still grows with the exponential coefficient $n = 2\ \text{s}^{-1}$ and is potentially effective on the experiment time duration of several minutes. We suggest that, nevertheless, this instability may be insignificant because of the fall of the particles—hence of the interface—relative to the fluid. The perturbation in the fluid is expected to behave like

$$A \exp\left[-(z - z_0)\frac{2\pi}{\lambda} + nt\right] \qquad [3]$$

and for estimating what perturbation is encountered by the moving interface we have to substitute $z = z_F(t)$, see [1]. We get

$$A \exp[-t/\tau + nt], \quad (1/\tau) = 2\pi V_{\text{Stokes}}/\lambda. \qquad [4]$$

The relevant parameter is $1/\tau$, i.e. the relative rate of escape of the particle from the zone affected by the instability wave (to be more accurate, a hindrance function $h(\epsilon)/(1-\epsilon)$ must multiply V_{Stokes} in the definition). We argue that *if this parameter is larger than* n the particle interface moves into quiescent fluid faster than the instability, therefore, this interference will not display the unstable behavior estimated at $z = z_0$. Indeed, in the experiment with the abovementioned data $(1/\tau)$ was $63\ \text{s}^{-1}$, considerably larger than n.

2.1.2. The layer of pure fluid. After the particles move away from z_0 as described abvove, the left behind pure fluid of density ρ_C may eventually pick up instability modes with $\lambda > e$, therefore, it will be quickly replaced with the upper fluid of density ρ_U. The actual quite complicated three-layer system—a thin buoyant layer of fluid C with fluid U above and with suspension below—may contain a wealth of instability effects, see Craik & Adam (1979) and Lister & Kerr (1989), which are not pursued here. We are mainly interested in estimating the maximal thickness, d_{max}, that the layer of pure fluid may attain before it is destroyed by instability.

The considerations of Kerr & Lister (1992), based on viscous fluid results valid for short wave instability modes ($\lambda \ll \lambda_{mg}$) indicate that $d_{max} \sim [B(\rho_D - \rho_C)/(\rho_U - \rho_C)]^{1/2}a$, where B is a constant, say 10. For the present case this gives $d_{max} \sim 0.02$ cm, which is formed in $t_1 \sim 0.3$ s. This can be considered the lower bound for this effect, as verified below.

For estimating the upper bound of d_{max} we consider the inviscid long waves instability modes, $\lambda \sim$ (dimension of container). According to the foregoing arguments, these modes are drastically cut off in the mixture domain. Hence the lower mixture layer can be considered a solid wall boundary for the long waves, so that the two-layer system bounded from below is a reasonable approximation for estimation of d_{max}. Instantaneously, the upper layer of fluid U is thick and practically unbounded. Below it the thickness d of the layer of pure fluid C is rather small, at least at initial time.

The instantaneous value of d does not influence the onset of the instability, but the rate of growth is

$$n(d, \lambda) = n(\infty, \lambda)(\rho_U + \rho_C)^{1/2}\left[\frac{1}{\rho_U + \rho_C \coth(2\pi d/\lambda)}\right]^{1/2}, \quad n(\infty, \lambda) = \left[\frac{2\pi}{\lambda}\frac{\rho_U - \rho_C}{\rho_U + \rho_C}g\right]^{1/2}, \qquad [5]$$

so that for very small d/λ a reduction of approximately $2\sqrt{\pi}\sqrt{d/\lambda}$ occurs as compared with the unbounded from below ($d = \infty$) case.

Consider the development from $d = 0$. In lack of disturbances the layer of pure fluid grows with constant velocity $\approx V_{Stokes}$. First, when the layer is very thin, the disturbances on the interface between fluids C and U grow slower than d but eventually a balance is achieved. Further increase of d causes the layer to be destroyed (convected upwards) and replaced by fluid U. Then a new "cycle" begins.

An instability wave of wavelength λ strongly destabilizes—and probably destroys—a layer of given d in a period $t_2 \approx B/n(d, \lambda)$. The requirement $t_2 = t_1 = d/V_{Stokes}$ yields:

$$d_{max} = \left[\frac{B}{\pi 9\sqrt{2}}\frac{1}{\nu}a^{3/2}g^{1/2}\frac{\rho_D - \rho_C}{\rho_C}\left(\frac{\rho_U + \rho_C}{\rho_U - \rho_C}\right)^{1/2}\right]^{2/3} \cdot (a\lambda^2)^{1/3}. \qquad [6]$$

For our system, taking $B = 10$ and $\lambda = 19.5$ cm (the largest dimension of the container's cross section) we get $d_{max} = 0.44$ cm and $t_1 = 6$ s. Shorter λ give smaller d_{max} and t_1, but when λ approaches $\lambda_{mg} = 0.17$ cm the inviscid analysis becomes invalid. (As a speculation, however, we substituted $\lambda = 0.17$ cm in [6] and obtained $d_{max} = 0.02$ cm and $t_1 = 0.3$ s.)

We conclude that the pure fluid layer left behind the settling particles can reach at most the thickness of several millimeters during a transient process of formation and destruction whose cycles take at most several seconds each. Such a thin and unsteady layer is apparently beyond the resolution of the referred experiments, which may explain why it has not been documented.

2.2. *The case $\rho < \rho_U$*

Next we ask a more difficult question: why does the interface become unstable when the bulk density of the mixture below it, $\rho = \epsilon\rho_D + (1 - \epsilon)\rho_C$, is smaller than ρ_U. Again, this feature is supported by experiments (although the details are less clear-cut than before). Here we can presently suggest only a rough qualitative hypothesis: if the wavelength of the maximal growth instability driven by $(\rho_U - \rho)$ is much larger than the interparticle distance, e, and the corresponding rate of growth coefficient is larger than the relative rate of escape of the particles, $(1/\tau)$, then the "micro" distinction between interstitial fluid and suspended particles is not relevant to instability development.

For example, a suspension of the abovementioned components but with $\epsilon = 0.022$ has $\rho = 1.049$, smaller than $\rho_U = 1.070$. In corresponding single phase fluids of equal viscosity, the value $(\rho_U - \rho)/(\rho_U + \rho) = 0.01$ drives an instability with $\lambda_{mg} = 0.26$ cm, $n_{mg} = 9.5\ \mathrm{s}^{-1}$. Since $\lambda_{mg}/e = 29$ and $n_{mg}/(1/\tau) = 5.3$ we can argue that the maximal growth modes "sees" the suspension as a uniform bulk of density ρ.† In this example ρ_U was larger than ρ by 2%. As the density difference decreases λ_{mg} increases but n_{mg} decreases, and vice versa. It is, therefore, difficult to speculate for

†The ratio $n_{mg}/(1/\tau)$ can be considered marginal. Actually, in most of the experiments smaller particles were used, which reduces both e and $(1/\tau)$ and makes the bulk properties argument more convincing.

which value of the density difference parameter this bulk consideration becomes invalid and by which criterion it should be replaced. Our attempts to contrive a simple-for-experiments system of overlain suspension that clearly violates that bulk hypothesis [i.e. with both λ_{mg}/e and $n_{mg}/(1/\tau)$ smaller than 1 while $\rho < \rho_U$] were disappointing. The encountered difficulty is that in such a system the particles are big and fall with large velocity therefore must be followed on long settling distances unless the interstitial fluid is very viscous.

3. CONCLUDING REMARKS

In a typical suspension of particles of density ρ_D in a continuous fluid of density ρ_C overlain by a heavy fluid of density ρ_U, when $\rho_U > \rho_C$ the instabilty modes corresponding to the difference $(\rho_U - \rho_C)$ at the interface are strongly restricted by the small interparticle distance, $e \approx 2a\epsilon^{-1/3}$, of the order of magnitude of 0.1 mm. The particles settle away from the perturbed interface region before these short and slow waves that form between the particles attain a big amplitude. The thickening pure fluid layer left behind the particles is restricted only by the wall of the large container and may pick up instability waves whose rate of growth increases with the thickness of this layer. Before this thickness reaches several millimeters the instabilities become so fast that the pure fluid in this layer is practically convected upwards, while heavy fluid replaces it. Averaging over these quick cycles, the suspension appears in contact with the fluid of density ρ_U.

If the bulk suspension density, ρ, is larger than ρ_U the interface particles move into a quiescent domain, therefore the interface and the suspension below it look stable. If $\rho < \rho_U$ long and fast instability waves driven by the difference $(\rho_U - \rho)$ are possible. On the length and time scales of these perturbations the suspension is "seen" as a bulk of uniform density and velocity.

Since the interparticle distance is proportional to $\epsilon^{-1/3}$ even quite dilute suspensions (say, $\epsilon = 10^{-4}$) are consistent with these considerations. This increases the confidence in applying the averaged continuum formulation in the analysis of non-concentrated suspensions, from an unusual angle of view. (The ususal justification is the large number of particles in the system, but this cannot be directly used in answering the questions discussed here.)

The term "typical suspension" is of course problematic, but we must use it because although theoretically the abovementioned features are not necessary we could not contrive an easy-for-experiment case that is expected to behave differently.

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